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Fluctuation-spectra of few- and large-degrees-of-freedom chaotic systems

Katsuhito Matsui[1], Syuji Miyazaki[1], Miki U. Kobayashi[2], Takehiko Horita[3] and Katsuya Ouchi[4]

[1] *Graduate School of Informatics, Kyoto University, Kyoto 606-8501, Japan*

[2] *Research Institute for Mathematical Sciences, Kyoto University, Kyoto 606-8502, Japan*

[3] *Department of Mathematical Sciences, Osaka Prefecture University, Osaka 599-8531, Japan*

[4] *School of Design, Kobe Design University, Kobe, 651-2196, Japan*

In few-degrees-of-freedom chaotic dynamical systems, local expansion rates which evaluate an orbital instability fluctuate largely in time, reflecting a complex structure in the phase space. Its average is called the Lyapunov exponent, whose positive sign is a practical criterion of chaos. There exist numerous investigations based on large deviation statistics in which one considers distributions of coarse-grained expansion rates (finite-time Lyapunov exponent) in order to extract large deviations caused by non-hyperbolicities or long correlations in the vicinity of bifurcation points [1]. It is recently shown[2] that the Lorenz system[3] has both hyperbolic and non-hyperbolic parameter regions by use of covariant Lyapunov vectors[4]. The Lorenz plot also reflect a difference between hyperbolicity and non-hyperbolicity as shown in Figs. 1(a) and 1(b). The chaotic attractors on the Poincaré section $z = r - 1$, on which two unstable equilibrium points satisfying $(\dot{x}, \dot{y}, \dot{z}) = (0,0,0)$ and $(x, y, z) \neq (0,0,0)$, are shown in Figs. 1(c) and 1(d). In these figures, the nullclines are also shown as straight lines satisfying $\dot{x} = \dot{y} = 0$ on the plane $z = r - 1$ and as hyperbolic lines satisfying $\dot{z} = 0$. The two equilibrium points are given by a pair of intersections of the nullclines. We see hook-shaped parts of the attractor in Fig. 1(e) are considered to be tangent points of stable and unstable manifolds in the vicinity of the unstable equilibrium points causing nonhyperbolicity. We will show that the fluctuation spectra (rate functions) of the local expansion rate distinguish the both. We also characterize spatio-temporal intermittency in a coupled systems of chaotic elements[5] and turbulence modeled by a shell model[6] using statistical properties of not only the largest but also all other Lyapunov exponents such as variances and rate functions as well as Lyapunov dimensions. Figure 1(f) depicts a spatio-temporal pattern showing fully developed spatio-temporal chaos obtained from a coupled system consisting of hundred identical chaotic logistic maps. The fluctuation spectra of the 5th, 25th, 35th, 55th and 85th Lyapunov exponents are shown in Fig. 1(g). The variances of the local expansion rate, which is proportional to the curvature around the average (the Lyapunov exponent) of the fluctuation spectrum, as a function of the Lyapunov exponent are plotted in Fig. 1(e). Although a parabolic dependence is observed in this case, a completely different average-dependence of variance is obtained in a shell model of turbulence.

References

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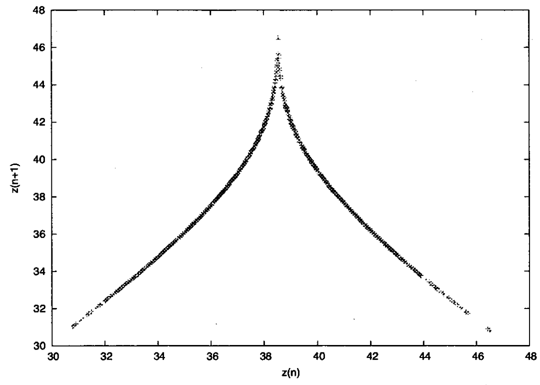
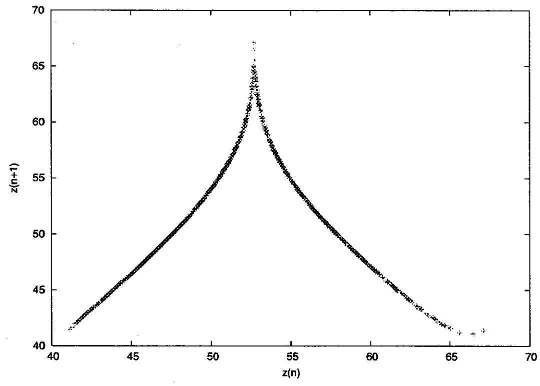
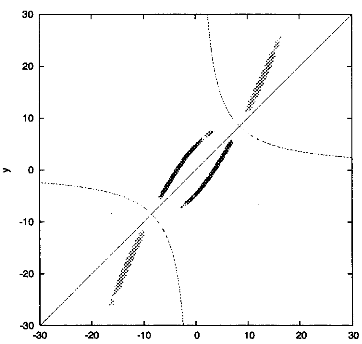


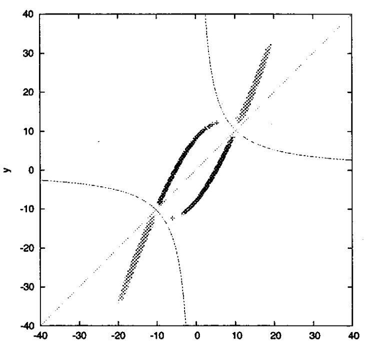
(a) $r = 28$ (hyperbolic)



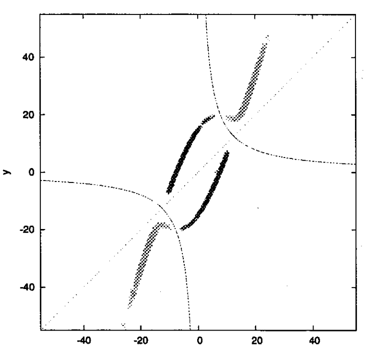
(b) $r = 40$ (non-hyperbolic)



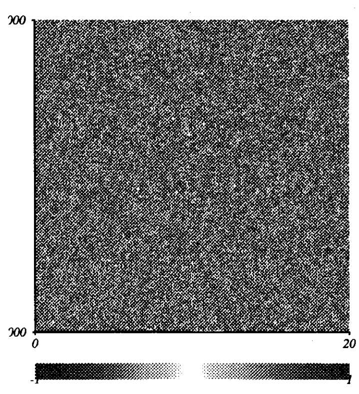
(c) $r = 28$ (hyperbolic)



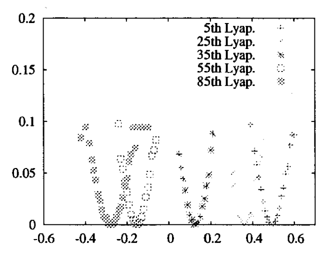
(d) $r = 40$ (non-hyperbolic)



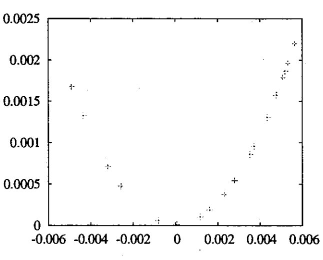
(e) $r = 60$ (non-hyperbolic)



(f)



(g)



(h)

Figure 1: (a,b) Lorenz plots and (c,d,e) attractors on the Poincaré section $z = r - 1$ of the Lorenz equation $(\dot{x}, \dot{y}, \dot{z}) = (-10x + 10y, -xz + rx - y, xy - 8z/3)$. (f) Spatio-tempral pattern, (g) rate functions of the local expansion rates, and (h) variances of the local expansion rates as a function of the average (the Lyapunov exponent) of the coupled map lattice in a fully developed spatio-temporal chaos.